

Basic Skills and Technology

- not a Contradiction, but a Completion

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Abstract

Are certain basic (manipulating and other) skills still necessary in math teaching? If so, then we should talk about which ones and in which amount. Some examples shall demonstrate that we can use modern computer and hand held technologies to encourage pupils and students of all levels to practise those skills which might be neglected in many cases using technology in math education. The examples presented have proved to be successful in classes but certainly there are many possibilities to improve the basic ideas in many ways using the features of all resources available at the moment.

At almost all discussions on the use of a CAS in teaching mathematics the question appears if mathematical basic (manipulating) skills are lost by using computers or not. At the same occasions it is emphasized by the members of the discussion that there is a need in investigating which basic skills are still necessary and in which amount.

I start from my point of view that - at this moment - we cannot and should not do without certain basic skills in mathematics. I feel unable to give any forecast for the future. I am sure that we could significantly rise the acceptance of technology in teaching mathematics by sceptic and critical teachers or technology refusing educational systems if we would be able to apply technology not only to present real life problems or new - for some people radical - didactical approaches but also to improve basic calculating and manipulating techniques and mathematical capabilities. We can very consciously practise all those techniques using technology which could be neglected by a (too?) intensive use of the technology.

It would be necessary and helpful to set up a list of such basic techniques which teachers would like to have practised by their students. From my own experience I can say that it is very useful to provide the pupils - or students - with tools which enable them to help themselves in case of troubles and difficulties. It is not sufficient to have a textbook with lots of examples followed by the solutions and the students don't find the way from the problem to the solution. At the other hand it is not sufficient to have problems followed by a step by step solution. Sometimes it could be useful to work out a student - computer interactive strategy to help overcoming some deficits.

The teachers should present their wishes and visions independent of any special soft- or hardware and without any need of producing ideas how to realise their ideas. The software people could present or describe potentialities which might be unknown by most of us teachers.

I collected a few ideas of skills which could be practised and improved using computers (that is only a glimpse and I know that the discussion could lead to the conclusion: "we don't need this or that any longer in computer age").

- Estimating numerical results
- Working with percentages
- Working with fractions
- Elementary algebraic operations (factorising,)
- Recognising types of functions by their graphs
- Functions (Relations) and their various representations
- Solving equations (linear, quadratics and systems of equations)
- Calculus techniques
- Improving 2D and 3D imagination
- Calculating mentally
- Transformations of functions
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Incited by ideas from J. Vermeulen (Belgium), Heinz Rainer Geyer (Germany) and J. Wiesenbauer (Austria) I tried to produce training- and practise programs for my students (secondary level II), which I want to present as seed for further developments. The platforms are DERIVE and the TI-92. The only reason for that is because I mainly use those to CAS. I am sure that you can use each of the available systems, but the goal must be to develop software products which will meet our - the teachers' and the students' - very special wishes, ideas and visions.

I have tried to work with such tools since long. In my first years working with computers I produced programs written in BASIC. It is quite nice to follow the development of hard- and software on one application: The Rule of Vieta, presenting random generated examples.

I will allow a nostalgic look into the program code – I am happy to have done this a couple of years ago. It helps a lot to program with modern software tools.

```

      V I E T A  -  ÜBUNGSPROGRAMM
      -----

Aufgabe 2                                davon richtig: 1

 $x^2 - 9x + 20 = 0$ 

Wie heißt die 1. Lösung  $x_1 = ?$  5
Wie heißt die 2. Lösung  $x_2 = ?$ 
leider falsch !

```

```

Datei Bearbeiten Ansicht Suchen Ausführen Debug Optionen Hilfe
VIETA.BAS
10 '**** Trainingsprogramm zum Vieta'schen Wurzelsatz
15 '*** Programmname M 1
20 '      Böhm 1985
25 COLOR 15, 1
30 RANDOMIZE TIMER
100 CLS : LOCATE 5, 20: PRINT "V I E T A - ÜBUNGSPROGRAMM"
110 LOCATE 6, 20: PRINT "-----"
115 AU = AU + 1
120 LOCATE 8, 10: PRINT "Aufgabe"; AU
130 LOCATE 8, 40: PRINT "davon richtig:"; R
135 FOR Z = 14 TO 22: LOCATE Z, 1: PRINT STRING$(75, " "): NEXT Z
140 T = 0
200 B = INT(RND(1) * 26) + 97
210 X1 = INT(RND(1) * 21) - 10: X2 = INT(RND(1) * 31) - 15
215 IF X1 * X2 = 0 OR X1 = -X2 THEN 210
220 B$ = CHR$(B)
225 K1 = -X1 - X2: K2 = X1 * X2
226 IF K1 < 0 THEN V1$ = "-" ELSE V1$ = "+"
- Direkt -
UMSCH+F1=Hilfe F6=Fenster F2=SUBs F5=Ausf F8=Schrift 00001:001

```

Then I changed to *DERIVE* working with a class in the computer lab:

#3: VIE(3, x)

#4:

$$\begin{bmatrix} x^2 - 49 = 0 \\ x^2 - 21 \cdot x + 108 = 0 \\ x^2 - 5 \cdot x - 6 = 0 \end{bmatrix}$$

#5:

$$H \begin{bmatrix} x^2 - 49 = 0 \\ x^2 - 21 \cdot x + 108 = 0 \\ x^2 - 5 \cdot x - 6 = 0 \end{bmatrix}$$

#6:

$$\begin{bmatrix} x = 7 & x = -7 \\ x = 9 & x = 12 \\ x = -1 & x = 6 \end{bmatrix}$$

Congratulations if you are right. Now try the next five problems!

Press any key to continue_

Simp(#5)
Free:100%
Derive Algebra

The idea stayed the same, the tool changed and it changed once more to handheld technology. I rediscovered my old BASIC-program, converted it to the TI's syntax and we practised "Vieta" in the lessons, in the breaks, at home, sometimes in the train or sometimes just for fun or – how a student told us – as a means to find her concentration before learning for other subjects.

Practise the Rule of Vieta

```

Algebra Calc Other PrgmIO Clear On/Off
Problems: 1, correct 1
 $g^2 + 18 \cdot g + 80 = 0$ 
1. Solution:
-10
2. Solution:
-8
right
End = ESC. next = any
MAIN DEG AUTO FUNC 0/30 13153

```

```

Algebra Calc Other PrgmIO Clear On/Off
Problems: 3, correct 3
 $c^2 - 10 \cdot c - 11 = 0$ 
1. Solution:
-11
2. Solution:
1
sorry, false x1 = 11, x2 = -1
End = ESC. next = any
MAIN DEG AUTO FUNC 0/30 13153

```

Allow me another remembrance to my BASIC past: a program to revise the students' knowledge on set theory. I am still using that tool, the students like it and it doesn't take too long time to achieve quite reasonable results.

```

A N G A B E :
G = {9,17,12,20,15,2,3,25,7,10,16,11,6,22}
A = {20,16,17,11,6,7,15,12,25,22,2,9,3,10}
B = {20,22,6,15,3,9,11,10}
C = {12,25,11,16}

```

Aufgabe : $(A \setminus B) \setminus (A \setminus C)$

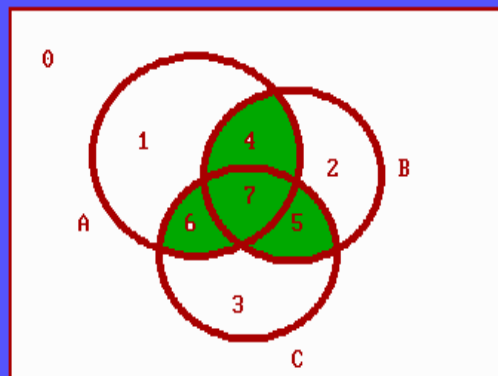
$(A \setminus B) \setminus (A \setminus C) = \{9,17,15\}$

Das stimmt leider nicht.

$(A \setminus B) \setminus (A \setminus C) = \{16,12,25\}$

< ENTER >

Problem : $(A \cup B) \cap (A \cup C) \cap (B \cup C)$



r i c h t i g ! < ENTER >

Für die Mengen nur den Buchstaben, für die Teilmengen nur die Ziffer eingeben.

Teilmengen werden grün gefüllt. Belegte Teilmengen können durch nochmaliges Ansprechen geleert werden. Die Eingabe kann abgeschlossen werden, wenn nur mehr dunkle Teile vorkommen.

? löst das Problem !

Some other examples are following:

Practise factorising (Jan Vermeylen)

#19: POL(3)

$$\begin{bmatrix} -2 \cdot x^2 - 16 \cdot x - \frac{158}{5} \\ 9 \cdot x^3 - 72 \cdot x^2 + 216 \cdot x \\ -6 \cdot x^3 + 84 \cdot x^2 - \frac{2052 \cdot x}{7} \end{bmatrix}$$

#20:

$$\begin{bmatrix} -2 \cdot \left[x + \frac{\sqrt{5}}{5} + 4 \right] \cdot \left[x - \frac{\sqrt{5}}{5} + 4 \right] \\ 9 \cdot x \cdot (x^2 - 8 \cdot x + 24) \\ -6 \cdot x \cdot \left[x + \frac{\sqrt{7}}{7} - 7 \right] \cdot \left[x - \frac{\sqrt{7}}{7} - 7 \right] \end{bmatrix}$$

#21:

#23: POL(4)

$$\begin{bmatrix} 9 \cdot x^4 - 18 \cdot x^3 + 9 \cdot x^2 \\ 9 \cdot x^2 - 27 \\ -63 \cdot x^3 - 318 \cdot x^2 + 807 \cdot x - 378 \\ 49 \cdot x^2 + 84 \cdot x + 36 \\ 36 \cdot x^2 + 96 \cdot x + 64 \end{bmatrix}$$

#24:

$$\begin{bmatrix} 9 \cdot x^2 \cdot (x-1)^2 \\ 9 \cdot (x + \sqrt{3}) \cdot (x - \sqrt{3}) \\ 21 \cdot (x+7) \cdot \left[\frac{9}{7} - x \right] \cdot (3 \cdot x - 2) \\ (7 \cdot x + 6)^2 \\ 4 \cdot (3 \cdot x + 4)^2 \end{bmatrix}$$

#25:

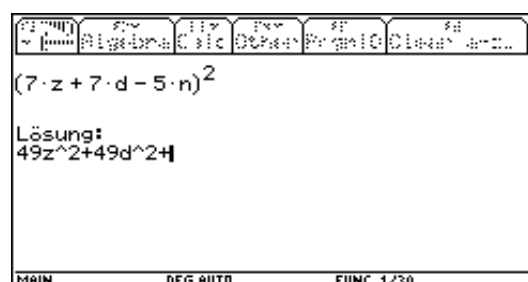
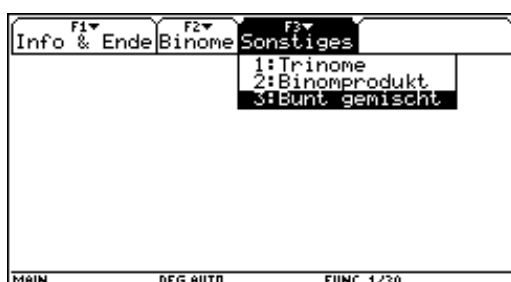
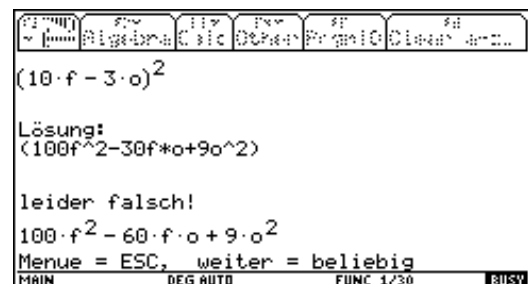
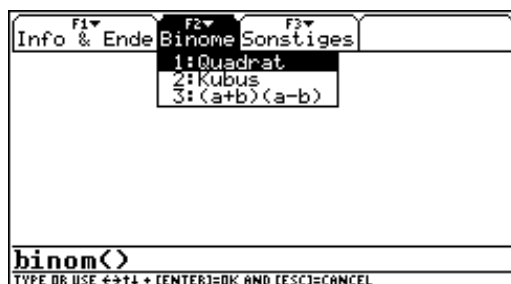
POL(n) produces n random number generated problems.

The students have to factorise using pencil and paper and then check their solution.

The file could easily be changed to practise special types of factorizations if necessary.

(This example is some years old and could be adapted to other platforms)

It could look like the next trainings tool (J.Böhm). My students liked it and used it:



They practised squaring and cubing binomials, squaring trinomials, multiplying binomials mentally and with "Bunt gemischt" they are presented a random generated sequence of problems. So you can have competitions in the class room and the pupils become very very busy, even the weaker ones.

The next example is from **H.R.Geyer** to help students improving their **mentally calculation skills**.

```

#27: SQUARES(5, 30) =
[ 9  "a1"  2 ]
[ 15 "a1"  2 ]
[ 11 "a1"  2 ]
[ 6  "a1"  2 ]
[ 4  "a1"  2 ]

#38: MIKE(10, 100) =
[ 1634 "a1"  38 ]
[ 1224 "a1"  36 ]
[ 37  "a1"  66 ]
[ 59  "a1"  55 ]
[ 16  "a1"  19 ]
[ 52  "a1"  70 ]
[ 86  "a1"  97 ]
[ 66  "a1"  70 ]
[ 42  "a1"  94 ]
[ 42  "a1"  88 ]

#39: CHECK [ 1634 / 38 = 43, 1224 / 36 = 34, 37 + 66 = 103, 59 - 55 = 4, 16 + 19 = 35,
[ 43 = 43 "right" ]
[ 34 = 34 "right" ]
[ 103 = 103 "right" ]
[ 4 = 4 "right" ]
[ 35 = 35 "right" ]
[ 3540 = 3540 "right" ]
[ -61 = -61 "right" ]
[ -4 = -4 "right" ]
[ 139 = 139 "right" ]
[ -46 = -46 "right" ]

COMMAND: MATH101 Build Calculus Solare Expand Factor Help Jump solve Manage
Options Plot Quit Renew Simplify Transfer Unarchive solve Window approx
Solve option G:\DEP\BME\BME97\MT Free:100% Derive Algebra
Simp(#39)

```

```

#22: SER(5)
[ 3 "de element is :" 22 ]
[ "het " "verschil is :" 19 ]
[ 3 "de element is :" 14 ]
[ "het " "verschil is :" 14 ]
[ 7 "de element is :" -103 ]
[ "het " "verschil is :" -19 ]
[ 14 "de element is :" -32 ]
[ 2 "de som is :" -39 ]
[ 10 "de element is :" 81 ]
[ 14 "de element is :" 113 ]

```

J. Vermeylen produced a DERIVE file to have lots of "standard problems" dealing with arithmetic series. "verschil" is the difference d of the series.

The other screen shot stems from a very busy discussion in the DERIVE community to produce a sort of report when solving an equation and applying various equivalence transformations. **Johann Wiesbauer** solved the problem with a very sophisticated tool.

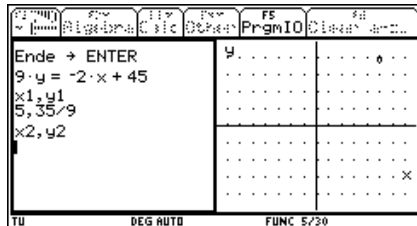
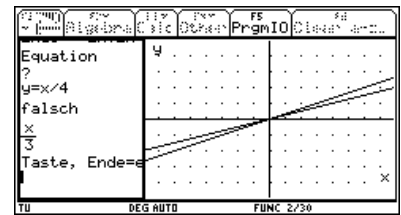
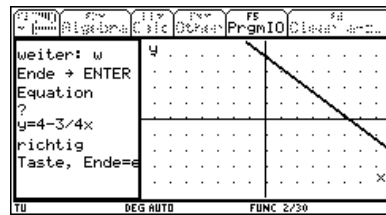
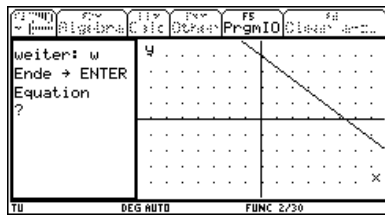
So students are able to reproduce, what they have done using paper and pencil and check their results with the computer's ones. They find their mistakes by themselves.

```

#16: DO(0 - 1) = (2 - x = 2)
#17: DO[0/2] = (x = 1)
[ 3^(2-x+1) - 7 = 20 [0 + 7] ]
[ 3^(2-x+1) = 27 [LN(0)] ]
[ LN(3) ]
#18: play = [ 2-x+1 = 3 [0 - 1] ]
[ 2-x = 2 [0/2] ]
[ x = 1 [1] ]
#19: 
#20: RECORD[ 2-x+1/3 = (1-4-x)/5 ] = [ 2-x+1/3 = (1-4-x)/5 ]
#21: DO(0-15) = (5-(2-x+1) = 3-(1-4-x))
#22: DO(0-15) = (10-x+5 = 3-12-x)
#23: DO(0-10-x) = (5 = 3-22-x)
#24: DO(0-3) = (2 = -22-x)
#25: DO[0/-22] = [-1/11 = x]
[ 2-x+1/3 = (1-4-x)/5 [15-0] ]
[ 5-(2-x+1) = 3-(1-4-x) [0-10-x] ]
[ 5 = 3-22-x [0-3] ]
[ 2 = -22-x [-0/22] ]
[ -1/11 = x [1] ]
#26: play =

```

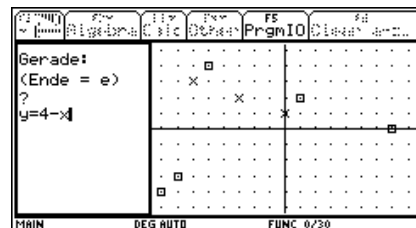
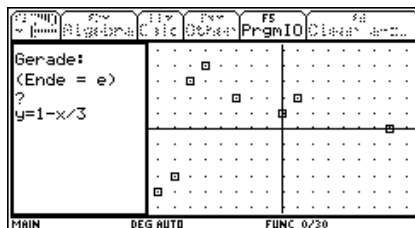
I found it useful and necessary as well that students can find the equation of a linear function, seeing its graph. They also should be able to have an imagination of the graph seeing the linear function very quick: "Match the Line!"



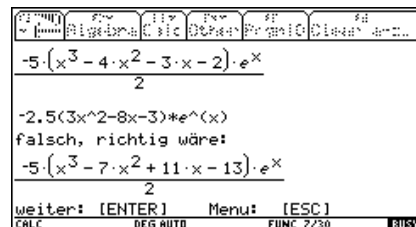
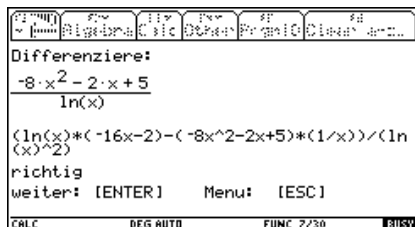
"Find two points!"

You enter the second point, see a second "ball" on the grid and if you are lucky the line passes the two "balls". The linear function appears in various representations (explicit, implicit, ..)

Finally I produce a "game". Give some random points on the grid and let the students hit them with as few lines as possible, or do that with parabolas



Another tool helped one of my classes to improve their differentiating and integrating skills. I produced a program calc() for the TI-92 and then they were able to practise whenever they had some time and what they found to be necessary for them:



There can be lots of improvements, I know. It would be better to explain the way how to do it correctly to have really a learning tool. What do you say to the next program, written by Philippe Fortin from France. He calls the program "stepder" from "Stepwise deriving functions":

Algebra	Calc	Deriv	Fract	Func	IO	Clear Up
Computing the derivative of						
$f(x) = x^3 \cdot \sin(x^3)$						
$f(x) = f1(x) \cdot f2(x)$						
$\frac{d}{dx}(f(x)) = \frac{d}{dx}(f1(x)) \cdot f2(x) + \frac{d}{dx}(f2(x)) \cdot f1(x)$						
Derivative of factor #1						
Computing the derivative of						
$f1(x) = x^3$						
STEPDER	RAD EXACT	FUNC 30/30	20195			

Algebra	Calc	Deriv	Fract	Func	IO	Clear Up
Computing the derivative of						
$f1(x) = x^3$						
$(x^n)' = n \cdot x^{(n-1)}$						
$\frac{d}{dx}(f1(x)) = 3 \cdot x^2$						
Derivative of factor #2						
Computing the derivative of						
$f2(x) = \sin(x^3)$						
STEPDER	RAD EXACT	FUNC 30/30	20195			

Algebra	Calc	Deriv	Fract	Func	IO	Clear Up
$\frac{d}{dx}(f1(x)) = 3 \cdot x^2$						
Derivative of factor #2						
Computing the derivative of						
$f2(x) = \sin(x^3)$						
It's a functions composition						
$f2(x) = \phi(u(x))$						
$\phi(x) = \sin(x)$						
$u(x) = x^3$						
STEPDER	RAD EXACT	FUNC 30/30	20195			

Algebra	Calc	Deriv	Fract	Func	IO	Clear Up
$\phi(x) = \sin(x)$						
$u(x) = x^3$						
The derivative of $\phi(u(x))$ is						
$u'(x) \cdot \phi'(u(x))$						
$\phi(x) = \sin(x)$						
The derivative of ϕ is						
$\frac{d}{dx}(\phi(x)) = \cos(x)$						
$f2(x) = \sin(u(x))$						
STEPDER	RAD EXACT	FUNC 30/30	20195			

Algebra	Calc	Deriv	Fract	Func	IO	Clear Up
Computing the derivative of						
$u(x) = x^3$						
$(x^n)' = n \cdot x^{(n-1)}$						
$\frac{d}{dx}(u(x)) = 3 \cdot x^2$						
then ...						
$\frac{d}{dx}(f2(x)) = 3 \cdot x^2 \cdot \cos(x^3)$						
STEPDER	RAD EXACT	FUNC 30/30	20195			

Algebra	Calc	Deriv	Fract	Func	IO	Clear Up
then ...						
$\frac{d}{dx}(f2(x)) = 3 \cdot x^2 \cdot \cos(x^3)$						
The derivative of						
$f(x) = f1(x) \cdot f2(x)$						
is then :						
$\frac{d}{dx}(f(x)) = 3 \cdot x^5 \cdot \cos(x^3) + 3 \cdot x^2 \cdot \sin(x^3)$						
End of the step by step computation.						
STEPDER	RAD EXACT	FUNC 30/30	20195			

The last example is part of a whole sequence to teach the pupil how they can help themselves reaching a sound level in working and manipulating with fractions.

Before working with fractions we had together developed a function to calculate the GCD of general expressions, to check the common denominator and then we used a selfmade function ew(expr) to find the expanding factor for the denominator expr with a common denominator gn.

Step by step the students can perform their calculation and check it using the calculator.

From the TI 92 Worksheet 3

We have predefined a function **ew(expr)** which returns the expanding factor for a fraction with denominator **expr** and the given common denominator **gn**

Train your skills in calculating and manipulating with fractions

Example 2: $\frac{16a + 37b}{4a + 10b} + \frac{6ab}{4a^2 - 25b^2} - 4 =$

Calculate without the TI. Write down your result:

Here it is.

Can you find a reason why this calculation could cause a very special mistake?

Compare the common denominator and the factors used to bring all the numerators over a common denominator.

Explain the last part in the edit line:

$$-4 * gn:$$

How to work with an expanding factor

$$ew() ? \quad -4 * ew(\dots).$$

The remaining work is very easy:

Explain the relation between left and right hand side in the last row of the history area:

The underlying basic idea sounds very strange and controversial:

In addition to all the wellknown reasons to include modern technologies into mathematics teaching, which I like and support in a very high degree, I also use the technology in many cases to improve manipulatings skills which seem to be no longer of any importance by using these technologies.

If you would like to try some of the tools presented in this paper, then please contact me. I would appreciate any suggestions and ideas dealing with this topic.

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